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# Nonlinear post-compression of a hybrid vortex mode in a gas-filled capillary

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**We demonstrate nonlinear temporal compression of a vortex beam by propagation in a gas-filled capillary. Starting from an ytterbium-based laser delivering 700  $\mu$ J 640 fs pulses at 100 kHz repetition rate, the vortex beam is generated using a spiral phase plate and coupled to a capillary where it excites a set of four modes that have an overlap integral of 97% with a Laguerre-Gauss  $LG_{10}$  mode. Nonlinear propagation of this hybrid, orbital angular momentum (OAM)-carrying mode results in temporal compression down to 74 fs at the output. Beam and pulse characterizations are carried out to determine the spatial profile and temporal duration of the compressed pulses. This result in multimode nonlinear optics paves the way towards the generation of OAM-carrying few-cycle pulses, isolated attosecond XUV pulses, and tunable UV pulses through resonant dispersive wave emission.**

<http://dx.doi.org/10.1364/ao.XX.XXXXXX>

## 1. INTRODUCTION

The existence of orbital angular momentum (OAM) of light and its transfer from light to matter was first demonstrated three decades ago [1]. An azimuthally varying, quantized phase,  $e^{i\ell\theta}$  is connected to photonic energy flow, implying a quantized, optical OAM in units of  $\ell\hbar$ . Such beams have a point of zero intensity in the transverse plane and are sometimes referred to as vortex beams. A natural basis for such beams are the Laguerre-Gaussian (LG) modes, in cylindrical coordinates

$$LG_{\ell p}(r, \theta, z) = LG_{00} L_p^{|\ell|} \frac{r^{2|\ell|}}{w^2} \left(\frac{r}{w}\right)^{|\ell|} e^{-i(|\ell|+2p)\zeta - i\ell\theta}, \quad (1)$$

where  $LG_{00}$  is the Gaussian beam,  $p \in \mathbb{N}$ ,  $\ell \in \mathbb{Z}$ ,  $w(z)$  and  $\zeta(z)$  are the Gaussian beam radius and Gouy phase, and  $L_p^{|\ell|}$  are generalized Laguerre polynomials.

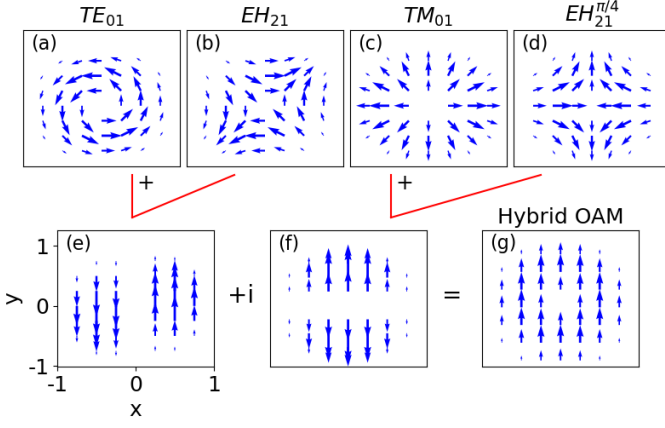
Femtosecond vortex beams have been gaining interest in the ultrafast community, for instance in atomic spectroscopy [2] and dichroism of chiral molecules [3, 4]. To this end, the transfer of

OAM from an infrared driver to the extreme ultraviolet through high harmonic generation was studied extensively [5, 6]. Recent studies show the potential of OAM beams to study chiral and/or topological magnetic materials using helicoidal dichroism in the EUV and X-ray domains [7].

Although structured light beams carrying OAM have been produced using optical elements like spiral phase plates (SPP)[8] for decades, these methods cannot be applied with few-cycle pulses because of chromatic effects. Another possibility to produce few-cycle vortices is to perform nonlinear temporal compression on a previously converted vortex beam, the generation of which would even allow the generation of isolated attosecond vortices [9]. In free space post-compression setups, the use of vortex beams was studied theoretically [10, 11] and demonstrated experimentally [12, 13] both for multi-plate schemes and multipass cells, in the context of energy scaling. In setups using guiding structures, although vortex beams have been extensively studied in optical fibers [14], there has been no demonstration of post-compression in a gas-filled capillary with a vortex beam. However it has been achieved with radially polarized beams [15]. Several recent studies also point towards exploiting the nonlinear propagation of multimode [16] or higher-order mode [17] fields in such guiding structures, and the field of multimode nonlinear photonics is expanding rapidly [18].

In this Letter, we show that it is possible to perform nonlinear temporal compression on a set of capillary modes synthesizing an  $LG_{10}$  free space mode. We first describe the hybrid OAM capillary modes to which an  $LG_{10}$  mode optimally couples before presenting the experimental results of spectral broadening with a guided vortex beam. This work paves the way to the generation of few-cycle OAM-carrying pulses, isolated attosecond vortices, and tunable deep and vacuum ultraviolet (V/DUV) OAM pulses through the resonant dispersive wave generation mechanism [19]. Using an optimized design based on a large core capillary, it could also be a technique for energy scaling of capillary-based compression setups.

The modes of a hollow-core capillary can be categorized into three families, namely the  $TE_{0m}$ ,  $TM_{0m}$  and  $EH_{nm}$  modes [20]. Similar to the case of optical fibers [14], it is possible to synthesize a field that is very close to an  $LG_{10}$  beam with linear polarization and a helical phase front by a linear combination of the  $TE_{01}$ ,



**Fig. 1.** Vector representation of the capillary modes that synthesize an OAM-carrying beam. Each vector represents qualitatively the intensity and direction of the electric field. (a)  $TE_{01}$  (b)  $EH_{21}$  (c)  $TM_{01}$  (d)  $EH_{21}^{\pi/4}$  (e) mode mixing of  $TE_{01} + EH_{21}$  and (f)  $TM_{01} + EH_{21}^{\pi/4}$ . The two modes in (e) and (f) when added with a phase shift of  $\pi/2$  yield the linearly polarized hybrid OAM mode (g).

$TM_{01}$ , and  $EH_{21}$  modes, as illustrated in Fig. 1.

$$\begin{aligned} OAM &= [TE_{01} + EH_{21}] + i [TM_{01} + EH_{21}^{\pi/4}] \\ OAM_x &= 0 \\ OAM_y &= 2J_1 \left( \frac{u_{11}r}{a} \right) e^{i\theta}, \end{aligned} \quad (2)$$

where  $J_1$  is the first-order Bessel function of the first kind,  $u_{11} \approx 3.83$  is its first zero, and  $a$  is the capillary radius. These capillary modes share the same propagation constant

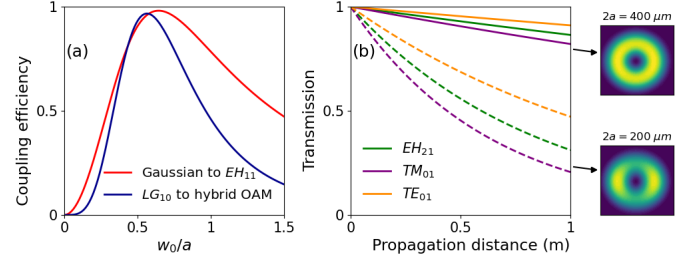
$$\beta = \frac{2\pi}{\lambda} \sqrt{1 - \frac{1}{2} \left( \frac{u_{11}\lambda}{2\pi a} \right)^2}, \quad (3)$$

while their attenuation coefficient ( $\alpha$ ) depends on the nature of the mode.

$$\alpha = \left( \frac{u_{11}}{2\pi} \right)^2 \frac{\lambda^2}{a^3} \begin{cases} \frac{1}{\sqrt{v^2-1}}, & \text{for } TE_{01} \\ \frac{v^2}{\sqrt{v^2-1}}, & \text{for } TM_{01} \\ \frac{\frac{1}{2}(v^2+1)}{\sqrt{v^2-1}}, & \text{for } EH_{21} \end{cases} \quad (4)$$

where  $v$  is the refractive index of the capillary glass, and  $\lambda$  is the wavelength. Note that it is also possible to synthesize left- or right-circularly polarized  $LG_{10}$ -like modes using combinations of only two modes  $EH_{21} + iEH_{21}^{\pi/4}$  or  $TE_{01} + iTM_{01}$ .

We now examine the coupling and propagation losses properties of the hybrid OAM mode. Fig. 2(a) shows the overlap integral between this mode and a  $LG_{10}$  beam at its waist as a function of the waist size. For reference, the coupling of a Gaussian beam to the  $EH_{11}$  mode is also shown. The calculated optimal coupling efficiency is 97.2% for a waist radius equal to  $0.56a$ , to be compared to 98.1% for a waist radius of  $0.64a$  in the Gaussian case. This shows that coupling to this hybrid mode can be almost as efficient as in the fundamental mode with a Gaussian beam. However, the coupling tolerance for variations in spot size is lesser for the  $LG_{10}$  case, making it experimentally more difficult to achieve.



**Fig. 2.** (a) Coupling efficiency as a function of the ratio of beam waist  $w_0$  to capillary core radius  $a$  between a perfect Gaussian beam and  $EH_{11}$  capillary mode (red) and between a  $LG_{10}$  beam and the hybrid OAM mode (blue). (b) Optical power transmission as a function of propagation distance for the four modes in a capillary of core diameter  $400 \mu\text{m}$  (solid curves) and  $200 \mu\text{m}$  (dashed curves), with the corresponding simulated intensity profiles at the output.

From Eq. (3) and Eq. (4), we can see that the modes that contribute to the hybrid OAM mode are degenerate in propagation constant, but not in attenuation coefficients. This difference in propagation losses modifies the intensity, phase, and polarization state pattern of the hybrid OAM mode at the output. For example, in Fig. 2(b), we plot the transmission of the three modes for capillary core diameters of  $400 \mu\text{m}$  and  $200 \mu\text{m}$ , at a wavelength of  $1.03 \mu\text{m}$ . Since the optical losses scale as  $1/a^3$ , they are both higher and spread over a larger relative interval for the smaller capillary. As a result, their effect on the beam output intensity profile is more visible when the core radius of the capillary is small, as also displayed in Fig. 2(b).

The OAM-carrying output field can be derived analytically by taking into account the different propagation losses. Simple algebra yields

$$|OAM_y(z)| = J_1 \left( \frac{u_{11}r}{a} \right) \left( e^{-\frac{X}{2}z} + e^{-\frac{X}{2}v^2z} \right) \times \sqrt{\cos^2 \theta e^{-Xz} + \sin^2 \theta e^{-Xv^2z}} \quad (5)$$

$$\arg [OAM_y(z)] = \arctan \left( \tan \theta e^{-\frac{X}{2}(v^2-1)z} \right) \quad (6)$$

$$|OAM_x(z)| = J_1 \left( \frac{u_{11}r}{a} \right) \left( e^{-\frac{X}{2}v^2z} - e^{-\frac{X}{2}z} \right) \times \sqrt{\cos^2 \theta e^{-Xv^2z} + \sin^2 \theta e^{-Xz}} \quad (7)$$

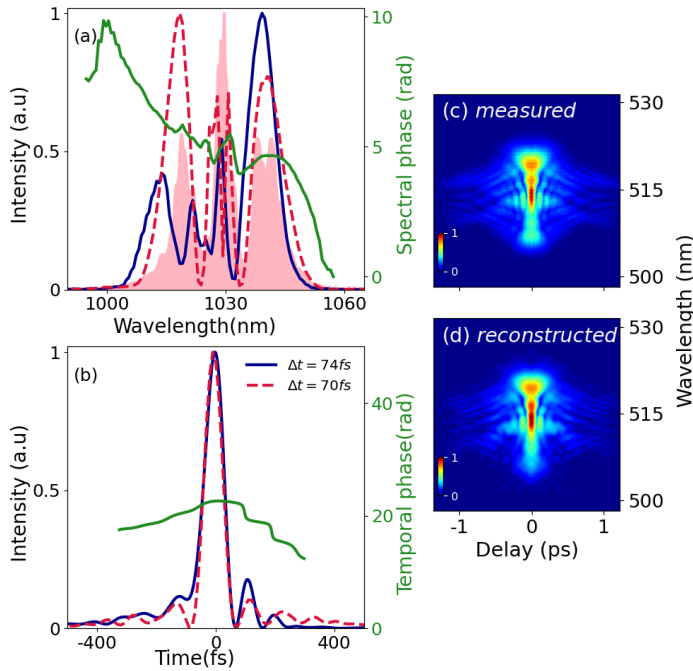
$$\arg [OAM_x(z)] = \arctan \left( \frac{1}{\tan \theta} e^{\frac{X}{2}(v^2+1)z} \right) \quad (8)$$

$$\text{with } X = \left( \frac{u_{11}}{2\pi} \right)^2 \frac{\lambda^2}{a^3} \frac{1}{\sqrt{v^2-1}}.$$

The deviation from the input in terms of intensity, phase, and polarization profiles increases as the core diameter decreases. But the fidelity of the output field is high in the case of high transmission, corresponding to most practical situations. This linear analysis of the modal properties leads us to use a capillary with a core diameter of  $400 \mu\text{m}$  in the experiments.

The schematic of the experimental set up is shown in Fig. 3. The experiment is carried out using a high-energy Ytterbium-based femtosecond laser source (Tangor, Amplitude) delivering  $700 \mu\text{J}$ ,  $640 \text{ fs}$  pulses at a central wavelength of  $1030 \text{ nm}$  and a repetition rate of  $100 \text{ kHz}$ , corresponding to an average power of  $70 \text{ W}$ . The Gaussian spatial mode at the output of the laser is





**Fig. 6.** Temporal characterization of the output compressed OAM-carrying pulses obtained with a capillary filled with 2.4 bar of Xe. (a) Spectral information. Shaded area: independently measured spectrum, blue curve: retrieved from FROG measurement, dashed red curve: simulation, green curve: spectral phase retrieved from FROG. (b) Temporal information. Blue curve: temporal intensity profile retrieved from FROG, dashed red curve: simulation. The legend shows the corresponding FWHM values. (c) Measured and (d) retrieved FROG traces.

with the fact that the effective area for the Gaussian and OAM-carrying beams can be approximated by  $A_{\text{eff}}^G = \pi \times (0.64a)^2$  and  $A_{\text{eff}}^{LG} = 2\pi \times (0.56a)^2$ , yielding a ratio of approximately 1.5. The coupling efficiency and propagation losses also affect the compression ratio. Estimated values of B-integral are 10 rad for the vortex beam and 17 rad for the Gaussian beam.

The FROG temporal characterization results of the compressed OAM-carrying pulse are shown in Fig. 6. The retrieved spectrum matches well the independently measured spectrum. The FTL of the retrieved spectrum is 74 fs, very close to the pulse duration of 70 fs obtained from simulations. These numerical simulations solve the vectorial multimode nonlinear Schrödinger equation including losses, dispersion, self-phase modulation, self-steepening, and ionization-related effects. The FROG-measured input laser intensity and phase profiles are set as the input conditions. The simulated spectrum and compressed pulse intensity profile are plotted in Fig. 6, with features in good agreement with the experimental FROG result. The energy contained in the main temporal lobe is estimated to be 200  $\mu\text{J}$ , 72% of the total available energy at the output.

In summary, we have demonstrated nonlinear post-compression of a hybrid vortex beam in a gas-filled capillary at a pulse energy level of 700  $\mu\text{J}$ . The spatial intensity and phase profile of the vortex beam are conserved upon nonlinear propagation, while ensuring a compression ratio of 9 for the chosen experimental parameters. This confirms the robust nature of the hybrid capillary mode for carrying a topological charge. In the

near future, such a setup combined with a first nonlinear pulse compression stage based on multipass cell could become a robust source for applications requiring few-cycle pulses carrying an OAM.

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**Disclosures.** The authors declare no conflicts of interest.

**Data Availability Statement.** Data underlying the results presented in this article are not publicly available at this time but may be obtained from the authors upon reasonable request.

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