



HAL
open science

Asymmetric comb waveguide for strong interactions between atoms and light

Nikos Fayard, A Bouscal, J Berroir, A Urvoy, T Ray, S Mahapatra, M Kemiche, Juan Ariel Levenson, Jean-Jacques Greffet, K. Bencheikh, et al.

► **To cite this version:**

Nikos Fayard, A Bouscal, J Berroir, A Urvoy, T Ray, et al.. Asymmetric comb waveguide for strong interactions between atoms and light. *Optics Express*, 2022, 30 (25), pp.45093-45109. 10.1364/OE.475162 . hal-03866287

HAL Id: hal-03866287

<https://iogs.hal.science/hal-03866287v1>

Submitted on 22 Nov 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Asymmetric comb waveguide for strong interactions between atoms and light

N. FAYARD,¹ A. BOUSCAL,² J. BERROIR,² A. URVOY,² T. RAY,² S. MAHAPATRA,³ M. KEMICHE,^{3,4} J. A. LEVENSON,³ J.-J. GREFFET,¹ K. BENCHEIKH,³ J. LAURAT,² AND C. SAUVAN^{1,*}

¹Université Paris-Saclay, Institut d'Optique Graduate School, CNRS, Laboratoire Charles Fabry, 91127 Palaiseau, France

²Laboratoire Kastler Brossel, Sorbonne Université, CNRS, ENS-Université PSL, Collège de France, 4 place Jussieu, 75005 Paris, France

³Université Paris-Saclay, CNRS, Centre de Nanosciences et de Nanotechnologies, 91120 Palaiseau, France

⁴Currently with Université Grenoble Alpes, Université Savoie Mont Blanc, CNRS, Grenoble INP,

IMEP-LAHC, 38000 Grenoble, France

*christophe.sauvan@institutoptique.fr

Abstract: Coupling quantum emitters and nanostructures, in particular cold atoms and optical waveguides, has recently raised a large interest due to unprecedented possibilities of engineering light-matter interactions. In this work, we propose a new type of periodic dielectric waveguide that provides strong interactions between atoms and guided photons with an unusual dispersion. We design an asymmetric comb waveguide that supports a slow mode with a quartic (instead of quadratic) dispersion and an electric field that extends far into the air cladding for an optimal interaction with atoms. We compute the optical trapping potential formed with two guided modes at frequencies detuned from the atomic transition. We show that cold Rubidium atoms can be trapped as close as 100 nm from the structure in a 1.3-mK-deep potential well. For atoms trapped at this position, the emission into guided photons is largely favored, with a beta factor as high as 0.88 and a radiative decay rate into the slow mode 10 times larger than the free-space decay rate. These figures of merit are obtained at a moderately low group velocity of $c/50$.

© 2022 Optica Publishing Group under the terms of the [Optica Open Access Publishing Agreement](#)

1. Introduction

The development of quantum technologies requires a number of prerequisites among which strong atom-photon interactions hold a high rank [1, 2]. Enhancing the interaction between a single photon and a single atom has been a driving force for a large community over the past decades. Two main routes have been followed rather independently: atoms in macroscopic high-finesse cavities [3, 4] or solid-state emitters (e.g., quantum dots) in micro and nanostructures [5, 6]. In recent years, various works have combined both approaches by interfacing cold atoms with nanophotonic devices such as photonic crystal nanocavities [7], nanofibers [8–10], and photonic crystal waveguides [11–13]. These hybrid strategies benefit from both the long coherence time of atoms and the enhanced electromagnetic field associated with subwavelength light confinement.

Besides the well-established field of cavity quantum electrodynamics (QED), the use of single-pass schemes with nanofibers or nanostructured waveguides has triggered the emergence of a new field of research known as waveguide QED [14–16]. Photons travelling in the waveguide carry the information through long distances while atoms can store it for long times. These systems are a promising building block for quantum networks as shown recently by the experimental demonstrations of a coherent photon storage [17, 18], the heralded creation of a single collective excitation of atomic arrays [19], or a correlated photon transport [20–22]. In addition, strong atom-photon interactions give rise to new phenomena in many-body physics [23] such as the emergence of solitons dynamics [24, 25] or many-body localization [26].

46 Most of the existing experiments in waveguide QED at optical frequencies are performed with
 47 silica nanofibers [8–10, 17–19, 22, 27–29]. They offer a 4π solid-angle access to the interaction
 48 region that markedly eases the manipulation of atomic clouds near the structure. In addition,
 49 nanofibers provide single-mode operation over a broad spectral range. It is thus possible to use
 50 guided light beams at frequencies detuned from the atomic transition for trapping the atoms at
 51 subwavelength distances from the waveguide [10, 30, 31]. However, the atom-photon interaction
 52 remains relatively weak. It can be quantified by the β factor, which is defined as the ratio of
 53 the radiative decay rate of a single atom in the waveguide mode Γ_{1D} to the total decay rate Γ_{tot} ,
 54 $\beta = \Gamma_{1D}/\Gamma_{\text{tot}} = \Gamma_{1D}/(\Gamma_{1D} + \Gamma')$. Photons that are not funnelled into the waveguide mode are
 55 lost in the radiation continuum with a decay rate Γ' . Experimentally observed values of β in
 56 nanofibers are typically in the range of $\beta \sim 0.05$ [32–34].

57 A promising route to increase Γ_{1D} and β is to use periodic dielectric waveguides, i.e., waveguides
 58 with a periodic modulation of the refractive index along the propagation direction. Indeed, the
 59 radiative decay rate of a single atom in a guided mode is given by $\Gamma_{1D}/\Gamma_0 = n_g\sigma/(2A_{\text{eff}})$, where
 60 Γ_0 is the atomic decay rate in free space, σ is the absorption cross-section, n_g and A_{eff} are
 61 the group index of the mode and the effective area at the atom position [11, 35]. In periodic
 62 waveguides, the coupling between contrapropagating modes results in the opening of bandgaps
 63 in the dispersion relation and in the apparition of band edges where the group velocity $v_g = c/n_g$
 64 goes to zero [36]. Close to these peculiar points, periodic waveguides support slow guided modes
 65 with large n_g 's, which, in turn, lead to increased values of Γ_{1D} and β [35]. Hence, an important
 66 research effort has been devoted to the design and the realization of periodic waveguides aimed
 67 at increasing the interaction with cold atoms [11–13, 37, 38]. However, this is a challenging task
 68 that requires tackling various issues [13, 39], including the sensitivity of slow light to fabrication
 69 imperfections, a stable trapping of atoms at the desired positions, and the accessibility around the
 70 structure to ease the transport of the atoms to the trapping sites.

71 In addition to enhanced atom-photon interactions, periodic waveguides also offer the possibility
 72 to create dispersion relations that greatly differ from the usual and almost linear dispersion of a
 73 nanofiber. The curvature of the dispersion relation between the frequency ω and the wavevector
 74 k has a profound impact on various physical phenomena, such as the distortion of short pulses
 75 during propagation, the collective properties of atomic arrays coupled to the waveguide, the
 76 localization of photons emitted in the bandgap [15], as well as the tolerance of the slow mode to
 77 inevitable fabrication imperfections [13, 40]. However, the engineering of the dispersion has
 78 been quite limited until now. Indeed, most of the periodic waveguides studied so far (be it for
 79 interacting with atoms or for another purpose) are made of a periodic pattern that is symmetric
 80 in transverse directions [11, 12, 36, 37, 41–47] and their dispersion relation varies most often
 81 quadratically at band edges.

82 By breaking the transverse symmetry, one can obtain new degrees of freedom for engineering
 83 the dispersion beyond the standard parabolic shape. Recently, Nguyen *et al.* demonstrated with a
 84 two-dimensional (2D) structure that symmetry breaking can be used to create exotic photonic
 85 dispersion relations such as Dirac cones, multivalley curves, or flat bands [48].

86 In this article, we exploit symmetry breaking to design a novel three-dimensional (3D) geometry,
 87 the asymmetric comb waveguide, that (i) supports a slow mode with a *quartic* dispersion, (ii)
 88 offers the possibility to trap atoms optically at subwavelength distances, and (iii) provides very
 89 large β factors for trapped atoms. We show that the quartic dispersion makes the slow mode more
 90 tolerant to fabrication imperfections than previous proposals [13]. The structure is represented
 91 in Fig. 1. It consists of a suspended bridge waveguide of width H and thickness t_y that has
 92 been periodically corrugated with an asymmetric rectangular pattern. This structure opens new
 93 opportunities in waveguide QED since it provides both a strong atom-photon interaction and an
 94 unusual dispersion that differs from the standard parabolic shape.

95 The article is organized as follows. In Section 2, we illustrate with 2D examples the role of

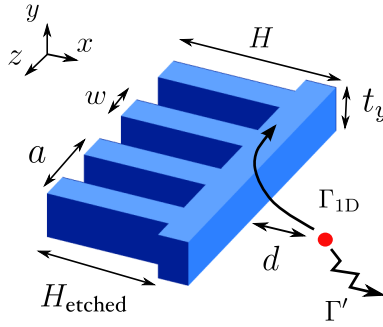


Fig. 1. Scheme of the asymmetric comb waveguide. The structure is etched in a GaInP membrane (refractive index $n = 3.31$) suspended in air. The waveguide has a total width H in the transverse x direction and a thickness t_y in the transverse y direction. The comb pattern along the propagation direction z is made of teeth with a width w and a height H_{etched} , periodically spaced with a period a . The width of the guiding region is thus $H - H_{\text{etched}}$. An excited atom (red dot) decays radiatively either into the guided mode, with a rate Γ_{1D} , or into the radiation continuum with a rate Γ' .

96 symmetry breaking in the design of a flat band with a quartic dispersion. Then, we design in
 97 Section 3 a 3D comb waveguide with a quartic dispersion after having detailed a few important
 98 criteria that should be fulfilled by a periodic waveguide aimed at interacting with cold atoms.
 99 We present in Section 4 the calculation of the potential of a two-color optical trap made with
 100 guided modes. We show that cold Rubidium (Rb) atoms can be trapped as close as 100 nm
 101 from the waveguide in a 1.3-mK-deep potential well by using relatively low powers ($P \sim 1$ mW)
 102 compatible with photonic structures. Finally, we calculate in Section 5 the emission rates Γ_{1D} ,
 103 Γ_{tot} , and Γ' of a trapped atom. We provide evidence for very large coupling between the atom
 104 and a slow mode with $n_g = 50$: $\Gamma_{1D} = 10\Gamma_0$, $\Gamma' = 1.3\Gamma_0$, and $\beta = 0.88$. Section 6 concludes the
 105 work and discusses some perspectives.

106 2. Slow mode with a quartic dispersion

107 In this Section, we show that symmetry breaking can be used to create a comb waveguide that
 108 supports a slow mode with a quartic dispersion around a zero-group-velocity point instead of
 109 the usual parabolic shape. In order to illustrate the impact of symmetry breaking, we compare
 110 the band diagrams of four different periodic waveguides with a period a . The waveguides are
 111 schematically represented in the insets of Figs. 2(a)-(d). All four are comb waveguides (see
 112 Fig. 1) with the same total width $H = 2a$ but different corrugations. The comparison is done
 113 with 2D structures ($t_y = \infty$) in TM polarization (magnetic field polarized in the y direction). We
 114 consider a refractive index $n = 2.85$ that corresponds to the effective index of the fundamental
 115 guided mode in a GaInP membrane of thickness 150 nm at $\lambda = 780$ nm.

116 The dispersion curves are calculated with a Bloch-mode solver developed for studying light
 117 scattering in periodic waveguides [49]. The solver calculates the wavevector k in the propagation
 118 direction as a function of the frequency ω . It is implemented with the aperiodic Fourier modal
 119 method (a-FMM) [50], which relies on an analytical integration of Maxwell's equations along the
 120 direction of periodicity (z axis) and on a supercell approach with perfectly-matched layers (PMLs)
 121 along the transverse directions (x and y axis). PMLs ensure a correct treatment of far-field
 122 radiation conditions; they are implemented as complex nonlinear coordinate transforms [51]. This
 123 numerical method is also used in the rest of the paper for the calculation of the field distributions
 124 and the decay rates.

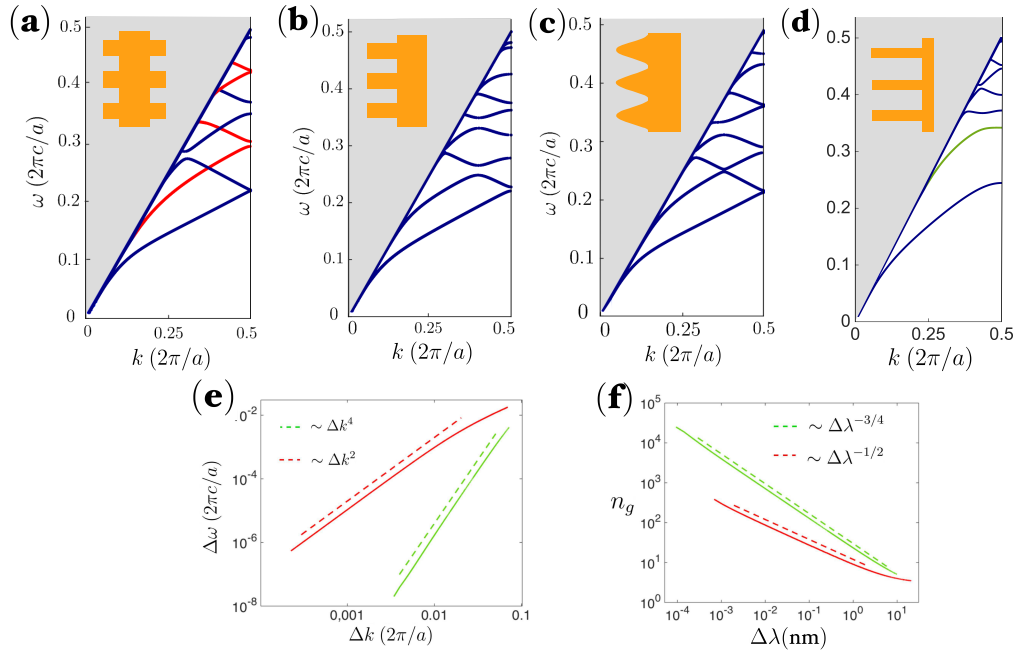


Fig. 2. Impact of a transverse symmetry breaking. (a)-(d) Band diagram $\omega = f(k)$ of four different 2D comb waveguides in TM polarization. The height of all structures is $H = 2a$, with a the period. To bring these 2D calculations closer to 3D simulations, we consider an effective refractive index $n = 2.85$. (a) Symmetric comb waveguide. The width of the teeth on both sides is $w = 0.5a$ and their depth is $H_{\text{etched}} = 0.25H$. (b) Asymmetric comb waveguide. The teeth are two times deeper, $H_{\text{etched}} = 0.5H$, with the same width $w = 0.5a$, to conserve the fraction of matter. (c) Asymmetric comb waveguide with a sinusoidal corrugation but the same fraction of matter. The depth of the cosine modulation is chosen so that the quantity of matter is the same as in (a)-(b). (d) Asymmetric comb waveguide designed to support a flat band with a quartic dispersion (green curve). The parameters are $w = 0.372a$ and $H_{\text{etched}} = 0.8H$. (e) Variation of $\Delta\omega = |\omega - \omega_e|$ as a function of $\Delta k = |k - \pi/a|$ in logarithmic scales (solid lines) for the green band in (d) and the red band of lowest frequency in (a). Dashed straight lines represent pure quadratic (slope of two in red) and quartic (slope of four in green) variations for comparison. (f) Group index as a function of $\Delta\lambda = |\lambda - \lambda_e|$. The quadratic and quartic dispersions lead to different scaling laws, as shown by the dashed straight lines that represent slopes of $-1/2$ (red) and $-3/4$ (green).

125 Figure 2(a) displays the band diagram of a comb waveguide whose corrugation is symmetric in
 126 the transverse direction. It can be easily understood with the following picture: the uncorrugated
 127 planar waveguide of width H supports symmetric and antisymmetric modes and the periodic
 128 corrugation couples forward and backward propagating modes, resulting in the opening of
 129 photonic bandgaps [36]. However, the symmetry of the corrugation forbids coupling between
 130 modes of different symmetry. Therefore, the Bloch modes resulting from the coupling can
 131 be sorted in two distinct families: they are either symmetric (blue curves) or antisymmetric
 132 (red curves). As can be seen, blue and red curves cross each other without coupling. Most
 133 of the bandgaps and the associated zero-group-velocity points are located at the edge of the
 134 first Brillouin zone, at $k = \pi/a$. Using a standard coupled-mode approach with two waves
 135 having linear dispersions, one can show that the dispersion relation of the Bloch modes varies
 136 quadratically in the vicinity of these points: $\omega - \omega_e \propto \pm(k - \pi/a)^2$, where ω_e is the frequency

137 of the band edge where $v_g = 0$ (see Supplement 1 for more details).

138 We now consider in Figs. 2(b)-(c) comb waveguides without mirror symmetry in the transverse
139 direction and with two different corrugations profiles. For the sake of comparison, the fraction
140 of matter (the orange area) is the same as in Fig. 2(a). The situation is now fundamentally
141 different: since the corrugations have no particular symmetry, all possible couplings are indeed
142 allowed. As a result, gaps are now open inside the Brillouin zone ($k \neq \pi/a$) around the points of
143 intersection between the dispersion curves of the symmetric and antisymmetric modes of the
144 uncorrugated waveguide. Looking precisely to the band diagrams, we observe that the spectral
145 widths of the bandgaps are smaller for a sinusoidal profile than for a rectangular profile. Thus,
146 the latter provides larger coupling strengths. This is the reason why we study the rectangular
147 asymmetric comb waveguide in the following.

148 For a rectangular profile, the coupling strengths depend on the size of the teeth, fixed by the
149 width w and the depth H_{etched} defined in Fig. 1. Stronger corrugations result in larger coupling
150 strengths and wider bandgaps. If the coupling is strong enough, two consecutive bandgaps may
151 overlap. In this case, the coupling mechanism is more complex since it involves four waves
152 instead of two [48]. This offers extra degrees of freedom for engineering the bands beyond the
153 standard parabolic shape.

154 Following this principle, we have designed an asymmetric comb waveguide that supports a
155 flat band with a *quartic* dispersion curve of the form $\omega - \omega_e \propto -(k - \pi/a)^4$. This flat band is
156 highlighted in green in Fig. 2(d). In order to confirm the quartic variation, we plot in Fig. 2(e)
157 $\Delta\omega = |\omega - \omega_e|$ as a function of $\Delta k = |k - \pi/a|$ with logarithmic scales. The solid green curve is
158 extracted from the green band in Fig. 2(d). It follows a quartic variation, parallel to the dashed
159 green straight line with a slope of four. In contrast, the symmetric comb waveguide of Fig. 2(a)
160 (solid red curve) supports modes with a quadratic dispersion, as shown by the dashed red straight
161 line that represents a slope of two.

162 A flat band with a quartic dispersion broadens the useful bandwidth of the slow mode, i.e.,
163 the bandwidth over which n_g is larger than a target value. Indeed, a quartic dispersion produces
164 a group index that scales with the wavelength as $\Delta\lambda^{-3/4}$ while the group index of a quadratic
165 dispersion scales as $\Delta\lambda^{-1/2}$. These scaling laws are derived in Supplement 1 and probed in
166 Fig. 2(f). Therefore, if one wants to work at a given group index, the quartic dispersion (green
167 curve) allows an operation at a larger $\Delta\lambda$, i.e., at a frequency further from the band edge. We will
168 see in the next Section that this increase of the useful bandwidth is important to improve the
169 tolerance of the slow mode to fabrication imperfections.

170 Let us end this Section by emphasizing that a quartic functional form for the dispersion relation
171 of a slow mode is uncommon, in particular in the context of waveguide QED where it has not
172 been studied so far. It is thus important to design a waveguide with a quartic dispersion and a
173 3D geometry compatible with the interaction with cold atoms trapped in the air cladding. We
174 present this design in the following Section.

175 **3. Design of a three-dimensional asymmetric comb waveguide**

176 We first detail a few important requirements that apply to a slow-light waveguide designed to
177 interact with cold atoms. Then, we present the design of a 3D asymmetric comb waveguide that
178 supports a slow mode with a quartic dispersion at the transition frequency of Rb atoms and an
179 evanescent field that extends far into the air cladding for an optimal interaction. In addition, we
180 pay particular attention during the design process to the presence of blue- and red-detuned modes
181 that can be used to create an optical two-color trap.

182 *3.1. Design requirements*

183 The design of a periodic waveguide aimed at increasing the coupling with cold atoms should
184 maximize the emission rate into the waveguide mode Γ_{ID} . Since the group index diverges at a

185 band edge, the first idea is to align the transition frequency of the atom with any band edge of the
186 photonic dispersion diagram. Unfortunately, this simple design cannot work for several reasons.

187 First, slow light is very sensitive to fabrication imperfections, resulting in backscattering,
188 radiation losses, and potentially light localization [40, 52–54]. Therefore, in a practical situation,
189 fabrication imperfections set an upper bound to the group index n_g that can be reached [55–57].
190 To improve the fabrication tolerance, X. Zhang *et al.* proposed to use parabolic dispersion curves
191 with large effective photon masses, $m_{\text{eff}} = (\partial^2\omega/\partial k^2)^{-1}$ [13]. Therefore, the design should not
192 only increase the group index, but also reduce the curvature of the dispersion relation.

193 A second issue is the value of the effective mode area at the position of the atom, or equivalently
194 the effective cross section of the photon seen by the atom. One should be careful not to lose with
195 an increased mode area what is gained with an increased group index. Hence, the design has
196 to find a trade-off between two opposite trends. On the one hand, we need a mode whose field
197 extends far into the air cladding, implying that it weakly interacts with the periodic pattern. On
198 the other hand, we need to control the group velocity and the curvature of the dispersion relation,
199 meaning that we need a mode that strongly interacts with the periodic pattern.

200 Finally, a third important challenge is to generate a stable optical trap for the atoms at
201 subwavelength distances of the waveguide and to be able to bring the atoms inside the trap.
202 A fully integrated trapping scheme can be achieved by using “fast” guided modes (i.e., with
203 standard values of n_g) at frequencies detuned from the atomic transition. For instance, red- and
204 blue-detuned modes can be used to create a two-color trap [31]. The design of the periodic
205 waveguide should thus ensure the presence of additional modes with adequate field profiles,
206 which spatially overlap with each other and with the slow mode.

207 As a whole, the design of a periodic waveguide with increased atom-photon interactions is a
208 complex task that should meet the following criteria:

- 209 1. a slow and single-mode operation at the transition frequency of the atom (large n_g),
- 210 2. a large fraction of the electric field in air outside the structure (small A_{eff}),
- 211 3. a large effective photon mass for an improved robustness to fabrication imperfections,
- 212 4. the existence of additional modes at frequencies detuned from the atomic transition for
213 trapping the atoms optically with low powers (few mW),
- 214 5. a clear access around the structure to ease the transport of the atoms to the trapping sites.

215 Up to now, two main geometries have been investigated. The first one, the alligator waveguide,
216 is composed of a tiny 250-nm-wide air slot symmetrically surrounded by two corrugated bridge
217 waveguides [11, 12, 38]. This geometry fulfills the first two criteria (large n_g and small A_{eff})
218 as well as the fourth one. However, it has been shown in [13] that the effective photon mass
219 of the slow mode is small, meaning that alligator waveguides are very sensitive to fabrication
220 imperfections in the slow-light regime. The fifth criterion is not met either since the atoms need
221 to be loaded in a narrow interacting region, which constitutes a major experimental challenge [58].
222 External trapping via side illumination has been considered [39, 59], but a stable trapping of
223 atoms *inside* the air slot has not been demonstrated. As a result, the value of β and the number N
224 of trapped atoms are limited to $\beta \sim 0.5$ and $N \sim 3$ [12, 60].

225 A different geometry has been proposed in [13] to address these limitations. It consists of
226 a hybrid-clad waveguide that combines two guidance mechanisms: total internal reflection on
227 one lateral side with a sharp sidewall and photonic bandgap on the opposite lateral side with a
228 two-dimensional photonic crystal. The hybrid-clad waveguide provides a flatter dispersion curve
229 than the alligator waveguide (50 times enhanced effective photon mass) and a 2π solid-angle
230 access to the interaction region. However, the possibility to use detuned guided beams for
231 trapping the atoms (fourth criterion) has not been investigated yet. On the one hand, the bandgap
232 of the photonic-crystal cladding sets important constraints on the spectral range available to place

233 additional guided modes that could be used for trapping. On the other hand, the photonic-crystal
 234 geometry provides various degrees of freedom for engineering the band diagram.

235 In the following, we design an asymmetric comb waveguide that fulfills all five criteria. In
 236 addition, the dispersion relation of the slow mode is quartic instead of quadratic. We show
 237 that, thanks to the quartic dispersion, the slow mode of the comb waveguide is more tolerant to
 238 fabrication imperfections than the mode of the hybrid-clad waveguide proposed in [13] that mostly
 239 follows a standard parabolic dispersion. To further limit the loss issues related to fabrication
 240 imperfections, we have chosen to use a moderately slow mode with $n_g = 50$, a value compatible
 241 with current fabrication processes [57]. We calculate in Section 5 that, despite this moderate
 242 slowdown of the light, we can achieve a strong atom/photon interaction.

243 3.2. Asymmetric comb waveguide

244 In order to benefit for a large refractive index contrast, we consider a GaInP membrane (refractive
 245 index $n = 3.31$) with a thickness $t_y = 150$ nm, as shown in Fig. 1. Starting from the 2D structure
 246 in Fig. 2(d), we have designed a 3D comb waveguide that supports a slow mode with a quartic
 247 dispersion $\omega - \omega_e \propto -(k - \pi/a)^4$ and a group index $n_g = 50$ at $\lambda_0 = 780$ nm, the wavelength
 248 of the $5S_{1/2} \leftrightarrow 5P_{3/2}$ transition of Rb atoms. The geometrical parameters are $a = 283$ nm,
 249 $H = 2a$, $H_{\text{etched}} = 0.8H$, and $w = 0.422a$. The band diagram of this comb waveguide is shown
 250 in Fig. 3(a); the slow mode with a quartic dispersion is highlighted in green. We consider the
 251 modes whose field components E_x , E_z , and H_y are symmetric with respect to $y = 0$.

252 Figures 3(b) and 3(c) display the distribution in the (x, z) plane of the dominant components
 253 of the electric field (E_x and E_z) at $\lambda_0 = 780$ nm. Note that the evanescent tail of the electric
 254 field extends far in the air cladding. On the side opposite the teeth, the longitudinal electric-field
 255 component E_z is maximum in front of the teeth, at $z \equiv a/2 \pmod{a}$. The positions of the maxima
 256 of the transverse electric-field component E_x are shifted by $a/2$, at $z \equiv 0 \pmod{a}$.

257 In addition to the slow mode, the band diagram should also provide two additional modes,
 258 red- and blue-detuned with respect to the atomic transition, that can be used to realize an optical
 259 two-color trap. These modes should lie below the light line and below the frequency of the
 260 electronic bandgap of GaInP ($\lambda = 680$ nm). The design of the structure has been performed by
 261 taking into account these additional constraints. The red- and blue-detuned modes are highlighted
 262 by the red and light blue dots in Fig. 3(a). The calculation of the trapping potential is presented
 263 in Section 4.

264 To ease the future implementation of the structure, it is important to keep in mind that
 265 nanostructured devices are never manufactured to their nominal specifications. Hence, real
 266 periodic waveguides are not exactly periodic and suffer from random fabrication imperfections.
 267 Slow light is particularly sensitive to these imperfections. It experiences random scattering as it
 268 propagates, resulting in backscattering, radiation losses, and possibly light localization [52–54].

269 In the usual case of a parabolic dispersion, the effective photon mass $m_{\text{eff}} = (\partial^2\omega/\partial k^2)^{-1}$ is
 270 the good figure of merit to be considered. It has been shown that a larger effective photon mass
 271 improves the tolerance of a slow mode to fabrication imperfections [13, 40]. According to these
 272 works, in the case of a non-parabolic dispersion, one must consider a second figure of merit in
 273 addition to the effective photon mass m_{eff} : the distance of the operation frequency to the band
 274 edge, $\Delta\omega = |\omega - \omega_e|$. To improve the tolerance to fabrication imperfections, both figures of
 275 merit should be increased: the effective photon mass *at the operation frequency* and the distance
 276 of the operation frequency to the band edge.

277 We have calculated m_{eff} and $\Delta\omega$ for the asymmetric comb. They are shown by the green
 278 curves in Figs. 3(d) and 3(e). For the sake of comparison, we have also calculated these figures
 279 of merit for the hybrid-clad waveguide proposed in [13], which is the most resilient to fabrication
 280 imperfections in the waveguide QED literature. Figure 3(d) displays the distance of the operation
 281 wavelength to the band edge, $\Delta\lambda = |\lambda - \lambda_e|$, as a function of the group index. Regardless of the

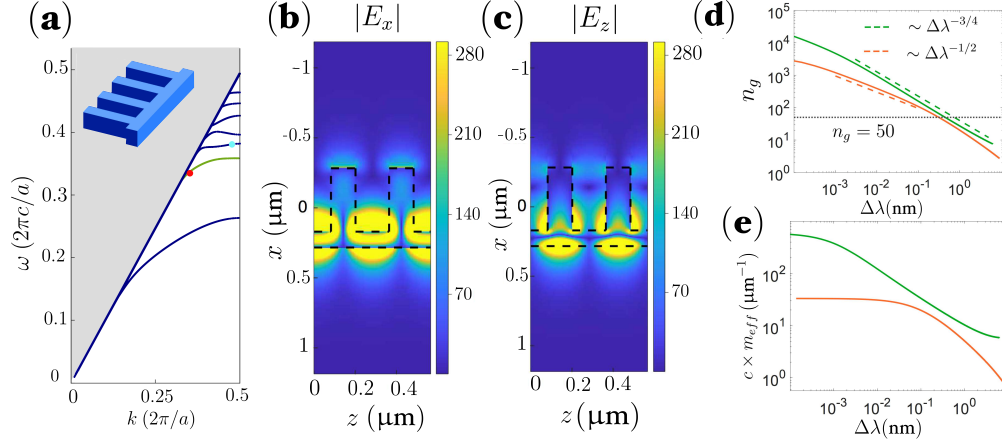


Fig. 3. Slow mode with a quartic dispersion. (a) Band diagram of the 3D comb waveguide designed to support a slow mode with a quartic dispersion (green curve) that strongly interacts with Rubidium atoms. The geometrical parameters are $a = 283$ nm, $H = 2a$, $H_{\text{etched}} = 0.8H$, $w = 0.422a$, $t_y = 150$ nm and $n = 3.31$. The red and light blue dots mark the “fast” modes red- ($\lambda_r = 837$ nm) and blue-detuned ($\lambda_b = 736$ nm) with respect to the atomic transition that are used for trapping, as detailed in Section 4. (b) Amplitude of the x -component of the slow mode electric field in $\text{V}/\mu\text{m}$. (c) Amplitude of the z -component of the slow mode electric field in $\text{V}/\mu\text{m}$. The field distributions are calculated for $\lambda_0 = 780$ nm at $y = 0$ in the middle of the membrane. (d) Group index as a function of the distance to the band edge, $\Delta\lambda = |\lambda - \lambda_e|$ (logarithmic scales). (e) Effective photon mass as a function of $\Delta\lambda = |\lambda - \lambda_e|$ (logarithmic scales). The green curves correspond to the slow mode of the 3D comb waveguide [green curve in (a)] and the orange curves correspond to the hybrid-clad waveguide in [13]. In (d), the dashed curves indicate the $\Delta\lambda^{-3/4}$ (green) and $\Delta\lambda^{-1/2}$ (orange) scaling laws.

282 value of the group index, the comb waveguide allows one to operate further from the band edge
 283 than the hybrid-clad waveguide. The dashed green and orange lines show that the asymmetric
 284 comb waveguide follows the scaling law that is expected for a quartic dispersion while the
 285 hybrid-clad waveguide mostly follows the scaling law of a quadratic dispersion. The dashed
 286 horizontal line marks the value $n_g = 50$ that is used in the following. Figure 3(e) shows the
 287 effective photon mass as a function of $\Delta\lambda$. Regardless of the operation frequency, the asymmetric
 288 comb has a larger effective mass than the hybrid-clad waveguide. Therefore, according to
 289 previous theoretical results [13, 40], the asymmetric comb waveguide should be more tolerant to
 290 fabrication imperfections than the hybrid-clad waveguide.

291 4. Two-color optical trap for Rubidium atoms

292 We show hereafter that the versatility of the asymmetric comb allows us to trap Rb atoms optically
 293 as close as 100 nm from the waveguide. At these deeply subwavelength distances, the atoms
 294 can strongly interact with the slow mode aligned with their transition frequency. The objective
 295 is to trap atoms where the electric field of the slow mode is intense, see Figs. 3(b)-(c). For
 296 accessibility reasons, it is not appropriate to create a trap between the comb teeth. Therefore, we
 297 aim at trapping atoms on the side opposite the teeth, see the red dot in Fig. 1.

298 There exist various ways to trap atoms close to a dielectric structure [30]. In waveguide QED,
 299 a smart, fully integrated, approach consists in taking advantage of the waveguide to build an
 300 optical trap with guided light beams. In this work, we use a two-color trap, whose principle is to
 301 send two additional light beams into the waveguide [31]. The first one is red detuned with respect

302 to the atomic transition; it produces a negative Stark shift on the energy of the ground state
 303 $5S_{1/2}$ that results in an attracting potential $U_r(x, y, z) < 0$. The second beam is blue detuned; it
 304 creates a positive Stark shift and a repulsive potential $U_b(x, y, z) > 0$. The combination of the
 305 two potentials,

$$U_{\text{trap}}(x, y, z) = U_r(x, y, z) + U_b(x, y, z), \quad (1)$$

306 can produce a deep potential well (i.e., a stable trap) at a given position in space that depends on
 307 the spatial profile of both guided beams, on their powers, and on the polarizability of the atom.

308 On top of the trapping potential U_{trap} generated by the detuned beams, it is necessary to take
 309 into account the attracting Casimir Polder (CP) potential that arises when an atom interacts with
 310 electromagnetic vacuum fluctuations near a dielectric surface [61, 62]. We assume that the CP
 311 potential felt by an atom close to the vertical sidewall of the comb (see the red dot in Fig. 1) is
 312 the same as the potential near a planar surface, which varies as $U_{\text{CP}} = -\frac{C_3}{d^3}$, where d is the distance
 313 between the atom and the surface and C_3 is a constant that depends both on the material and the
 314 atom. The value of C_3 for GaInP and Rb atoms, $C_3 \approx 6.7 \times 10^4 \text{ mK}\cdot\text{nm}^3$, is computed with the
 315 formula in [63]; the permittivity of GaInP is taken from the experimental data in [64]. The desired
 316 depth of the trapping potential for cold atoms is typically of the order of 1 mK. In comparison,
 317 the CP potential is negligible for distances larger than 100 nm, $U_{\text{CP}}(100 \text{ nm}) \approx -6.7 \times 10^{-2} \text{ mK}$,
 318 and it is dominant close to the surface $U_{\text{CP}}(10 \text{ nm}) \approx -67 \text{ mK}$.

319 In order to realize a two-color trap with guided modes, the frequencies of the detuned signals
 320 have to be chosen with the band diagram in Fig. 3(a). Since the waveguide is periodic, we
 321 are dealing with Bloch modes whose field distributions are periodically modulated along the z
 322 direction. It is thus possible to create a periodic array of potential wells in the z direction without
 323 using contra-propagating beams. It is convenient to choose the red-detuned frequency on the
 324 same band as the slow mode (green band), but closer to the light line where the group velocity
 325 is of the order of c/n . Then, one of the higher-order bands can be used for the blue-detuned
 326 frequency.

327 Since the wavelength of the $5S_{1/2} \leftrightarrow 5P_{3/2}$ transition of Rb atoms is $\lambda_0 = 780 \text{ nm}$, we have
 328 chosen to work with $\lambda_r = 837 \text{ nm}$ for the red-detuned field and $\lambda_b = 736 \text{ nm}$ for the blue-detuned

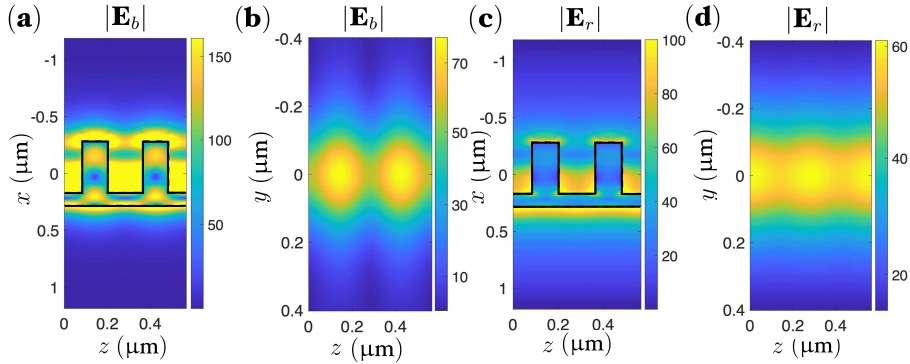


Fig. 4. Electric-field distributions of the blue- and the red-detuned guided modes used for the two-color trap. (a)-(b) Blue-detuned mode \mathbf{E}_b at $\lambda_b = 736 \text{ nm}$, marked by a light blue dot in the band diagram of Fig. 3(a). (c)-(d) Red-detuned mode \mathbf{E}_r at $\lambda_r = 837 \text{ nm}$, marked by a red dot in the band diagram of Fig. 3(a). In (a) and (c) the field is represented in the (x, z) plane at $y = 0$ (cross-section through the center of the GaInP membrane). The solid lines show the contour of the comb waveguide. In (b) and (d) the field is represented in a (y, z) plane located in air at a distance $d = 100 \text{ nm}$ from the structure. The fields are expressed in $\text{V}/\mu\text{m}$.

329 field. These wavelengths are displayed in Fig. 3(a) with red and light blue dots. The electric
 330 fields of the corresponding guided modes are shown in Fig. 4. The amplitude of the red-detuned
 331 mode is almost homogeneous in air along the z direction while the amplitude of the blue-detuned
 332 mode exhibits a more pronounced periodic modulation, with intensity maxima in front of the
 333 comb teeth at $z \equiv a/2 \pmod{a}$. Therefore, an atom near the back of the comb will “feel” an
 334 attracting potential that is almost independent of z and a repulsive potential that is stronger in
 335 front of the teeth. By playing with the relative powers of both detuned beams, it is possible to
 336 create a potential well with a minimum at $z \equiv 0 \pmod{a}$. An atom trapped at this position will
 337 interact efficiently with the slow mode since the transverse component of its electric field is
 338 maximum, see Fig. 3(b).

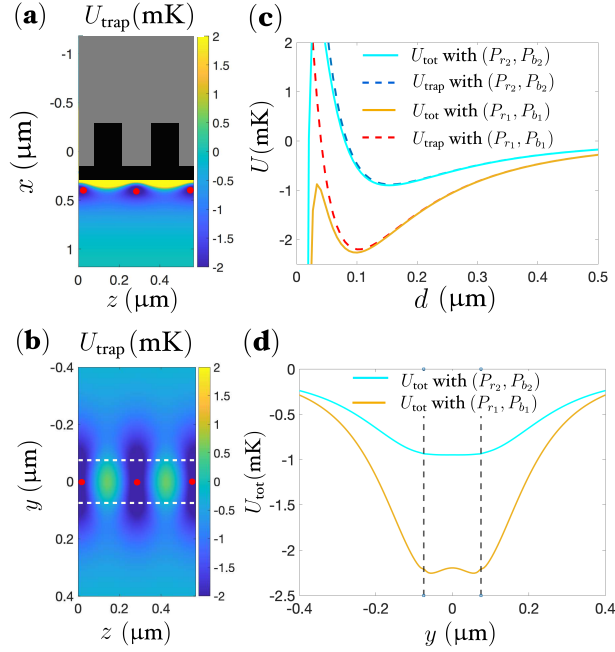


Fig. 5. Potential of the two-color trap. (a)-(b) Distribution of the trapping potential U_{trap} created by the superposition of the blue- and red-detuned modes. As in Fig. 4, we represent a cross-section in the (x, z) plane at $y = 0$ (a) and a cross-section in a (y, z) plane located in air at a distance $d = 100$ nm from the waveguide (b). Red dots represent atoms trapped in the potential wells. The black area in (a) shows the comb waveguide where the potential is not defined. We do not show the potential around the teeth (gray area); it is repulsive and does not exhibit trapping sites. The dashed lines in (b) remind the position of the GaInP membrane that is located in a different plane, shifted by 100 nm in the x direction. (c) Trapping potential U_{trap} (dashed curves) and total potential $U_{\text{tot}} = U_{\text{trap}} + U_{\text{CP}}$ (solid curves) as a function of the distance d to the waveguide for $y = 0$ and $z = 0 \pmod{a}$. Two different sets of trapping powers are represented. The powers $(P_{r_1}, P_{b_1}) = (1.6, 1.3)$ mW create a potential well at $d = 100$ nm with a depth $U \approx -2.2$ mK. The powers $(P_{r_2}, P_{b_2}) = (1, 1.3)$ mW create a potential well at $d = 160$ nm with a depth $U \approx -0.8$ mK. (d) Total potential U_{tot} as a function of the vertical coordinate y for $z = 0 \pmod{a}$. The orange (resp. light blue) curve shows the potential created at a distance $d = 100$ nm (resp. $d = 160$ nm) by blue- and red-detuned modes with powers (P_{r_1}, P_{b_1}) [resp. (P_{r_2}, P_{b_2})].

339 We have computed the trapping potential $U_{\text{trap}}(x, y, z)$ for Rb atoms with the recently-introduced
 340 open-source package Nanotrappy [65]. The inputs of Nanotrappy are the electric fields of the

341 blue- and red-detuned guided modes, which are calculated with the mode solver for periodic
 342 waveguides used to calculate the band diagram [49]. Nanotrappy calculates the Stark shifts
 343 induced by these electric fields and the corresponding potentials. The calculation includes the
 344 scalar, vector, and tensor shifts [65].

345 Figures 5(a)-(b) display maps of the trapping potential U_{trap} in the (x, z) plane at $y = 0$ and
 346 in a (y, z) plane located in air at a distance $d = 100$ nm from the structure. The powers of the
 347 red- and blue-detuned modes are respectively $(P_{r_1}, P_{b_1}) = (1.6, 1.3)$ mW. The potential is given
 348 in mK so that it can be easily compared with the temperature T of cold Rb atoms in a typical
 349 experiment, $T \sim 10$ μ K. Red dots highlight the potential minima, where atoms can be trapped, at
 350 $y = 0, z = 0 \pmod{a = 283 \text{ nm}}$, and a distance $d = 100$ nm from the structure. The confinement
 351 is good in all three directions of space.

352 Finally, we investigate in Figs. 5(c)-(d) the impact on the trap of the powers of the detuned
 353 modes, as well as the effect of taking into account the CP potential. Cross-sections of U_{trap}
 354 (dashed curves) and $U_{\text{tot}} = U_{\text{trap}} + U_{\text{CP}}$ (solid curves) are plotted for two different sets of trapping
 355 powers $(P_{r_1}, P_{b_1}) = (1.6, 1.3)$ mW and $(P_{r_2}, P_{b_2}) = (1, 1.3)$ mW. Note that the power of the
 356 blue-detuned mode (i.e., the repulsive potential U_b) is kept constant. Decreasing the power of
 357 the red-detuned mode has two main effects. First, the depth of the potential well is reduced from
 358 ≈ 2.2 mK to ≈ 0.8 mK. Second, the trapping distance is increased from 100 nm to 160 nm.
 359 Adding the CP potential does not alter the potential well. It is only modified at shorter distances
 360 where the attracting CP interactions become dominant. For the closest trap ($d = 100$ nm,
 361 orange solid curve), the potential barrier between the trap and the structure is lowered by the
 362 CP interactions but it remains higher than 1 mK. The orange curve in Fig. 5(d) shows that the
 363 potential minimum is not necessarily located at $y = 0$. The height of the tiny potential barrier at
 364 $y = 0$ is only 50 μ K

365 We have demonstrated the possibility to trap cold Rb atoms at deeply subwavelength distances
 366 ($d = 100$ nm) from the comb waveguide where the x -component of the electric field of the slow
 367 mode is intense. We have used a two-color optical trap with powers of about 1 mW, a value
 368 well compatible with the powers used in current integrated-optics experiments. We show in the
 369 following Section that an atom trapped at this position interacts strongly with the slow mode.

370 5. Strong atom-photon interaction

371 Having established the possibility to trap Rb atoms close to the comb waveguide, we now show
 372 that trapped atoms can strongly interact with the slow mode. For that purpose, we compute the
 373 spontaneous emission rate of an excited atom and the β factor – i.e., the fraction of light coupled
 374 to the slow mode.

375 With the optical trap discussed in Section 4, atoms are located at positions where the x -
 376 component of the electric field of the slow mode is dominant. Therefore, we consider in the
 377 following spontaneous emission of an atom with a dipole \mathbf{d} that is linearly polarized along the x
 378 direction, $\mathbf{d} = d\mathbf{e}_x$ with \mathbf{e}_x the unitary vector along the x direction. Experimentally, this situation
 379 can be realized by imposing a quantization axis along x with an external magnetic field.

380 We calculate the total emission rate Γ_{tot} and the emission rate in the slow mode Γ_{1D} (sum of
 381 the forward and backward propagation directions). The former is proportional to the imaginary
 382 part of the total Green tensor at the position \mathbf{r}_0 of the atom [66],

$$\frac{\Gamma_{\text{tot}}}{\Gamma_0} = \frac{6\pi c}{\omega_0} \mathbf{e}_x \cdot \text{Im} [\mathbf{G}(\mathbf{r}_0, \mathbf{r}_0, \omega_0)] \mathbf{e}_x, \quad (2)$$

383 while the latter depends only on the electric field of the guided mode [11, 35],

$$\frac{\Gamma_{\text{1D}}}{\Gamma_0} = \frac{n_g \sigma_a \varepsilon_0 |\mathbf{e}_x \cdot \mathbf{E}_m^*(\mathbf{r}_0)|^2}{2 \iiint_{\text{Cell}} \varepsilon |\mathbf{E}_m|^2 d^3 \mathbf{r}}. \quad (3)$$

384 In these expressions, ω_0 is the transition frequency, $\Gamma_0 = \omega_0^3 |\mathbf{d}|^2 / (3\pi\hbar\epsilon_0 c^3)$ is the emission
385 rate in vacuum, $\sigma = 3\lambda_0^2 / (2\pi)$ is the absorption cross-section, \mathbf{E}_m is the electric field of the
386 slow mode, and $n_g = c/v_g$ is its group index. One usually defines the mode effective area as
387 $A_{\text{eff}}(\mathbf{r}_0) = \iiint_{\text{Cell}} \epsilon |\mathbf{E}_m|^2 d^3\mathbf{r} / (a\epsilon_0 |\mathbf{e}_x \cdot \mathbf{E}_m^*(\mathbf{r}_0)|^2)$. Note that the volume integral runs over one
388 unit cell of the periodic waveguide.

389 Then, we deduce the emission rate outside the slow mode (i.e., into all the other modes),
390 $\Gamma' = \Gamma_{\text{tot}} - \Gamma_{1D}$, as well as the value of the β factor, $\beta = \Gamma_{1D} / \Gamma_{\text{tot}}$. Let us recall that the slow
391 mode supported by the comb waveguide designed in Section 3 has a group index $n_g = 50$ at the
392 transition wavelength of Rb atoms, $\lambda_0 = 2\pi c / \omega_0 = 780$ nm. Its electric field is represented in
393 Figs. 3(b)-(c).

394 The main difficulty is the calculation of the total Green tensor $\mathbf{G}(\mathbf{r}_0, \mathbf{r}_0, \omega_0)$ of an infinitely
395 long periodic waveguide, which requires an accurate calculation of the emission into radiation
396 modes with outgoing-wave conditions in a periodic medium [49]. Two different approaches
397 are often used to bypass this difficulty. The first one amounts to assume that the emission rate
398 into radiation modes is approximately equal to the emission rate in vacuum, $\Gamma' \approx \Gamma_0$, see for
399 instance [13]. With this approximation, Eq. (2) simply becomes $\Gamma_{\text{tot}} / \Gamma_0 \approx 1 + \Gamma_{1D} / \Gamma_0$. In that
400 case, the calculation only requires the knowledge of the guided mode, which can be calculated
401 with a Bloch-mode solver. The second approach that avoids the calculation of the Green tensor
402 of an infinitely long periodic waveguide consists in considering a finite-size structure formed
403 by a finite number of periods, see for instance [37]. The main drawback is that the calculated
404 structure (a Fabry-Perot cavity) is different from the desired one (an infinitely long waveguide).
405 As a result, the emission rate exhibits a series of spurious resonance peaks that depends on the
406 arbitrary choice of the structure length and termination. It is thus difficult to infer the actual
407 emission rate of the periodic waveguide.

408 In contrast to these two approximate approaches, we calculate rigorously the Green tensor of the
409 periodic waveguide by using a modal method that relies on an exact Bloch-mode expansion [49].
410 Figure 6 shows the variation of the decay rates Γ_{tot} , Γ_{1D} , Γ' (first line), and of the β factor (second
411 line) as a function of the position of the atom. In Fig. 6(a), the atom is moved horizontally
412 away from the comb for $y = 0$ and $z = 0 \pmod{a}$. In Fig. 6(b), the atom is moved vertically
413 for $d = 100$ nm and $z = 0 \pmod{a}$. In Fig. 6(c), the atom is moved along the waveguide for
414 $d = 100$ nm and $y = 0$. The yellow areas represent the volume where atoms are likely to be
415 trapped, defined as the positions where the value of the orange potential in Fig. 5 is between
416 $U_{\text{min}} \approx -2.2$ mK and $U_{\text{min}} + 50$ μ K. This defines a trapping volume of typical size 30 nm along
417 x , 160 nm along y and 20 nm along z .

418 For large distances, $d > 400$ nm, the emission rate Γ_{1D} into the slow mode is negligible
419 compared to the emission rate Γ' into the radiation continuum and the β factor tends towards zero.
420 As the distance d decreases, the atom enters the region where the field of the slow mode is intense
421 and Figs. 6(a) shows a strong enhancement of Γ_{1D} that results in an increase of the β factor. For
422 a trapping distance of $d = 100$ nm (yellow area), $\Gamma_{1D} = 10\Gamma_0$, $\Gamma' = 1.3\Gamma_0$, and $\beta = 0.88$. This
423 number is significantly larger than the value of $\beta \approx 0.5$ that has been experimentally observed for
424 the alligator waveguide [12, 60].

425 In Figs. 6(b)-(c), we fix the distance $d = 100$ nm and we vary the position of the atom in the
426 two other directions. Along the vertical direction, as long as the atom is located inside the trap
427 in front of the structure (-75 nm $\leq y \leq 75$ nm), the decay rate into the slow mode remains
428 dominant $\Gamma_{1D} > 8\Gamma_0$ and $\beta > 0.8$. Along the z direction, Γ_{1D} varies periodically. Its variation
429 over one period is directly related to the variation of $|E_x(z)|$ represented in Fig. 3(b). On the
430 other hand, Γ' is almost constant.

431 Finally, let us emphasize that the presence of the comb increases the emission rate into the
432 radiation continuum Γ' , compared to the emission rate in vacuum Γ_0 . Indeed, for $d \leq 100$ nm
433 and -75 nm $\leq y \leq 75$ nm, $\Gamma' \geq 1.3\Gamma_0$. Therefore, the assumption $\Gamma' \approx \Gamma_0$ would lead to an error

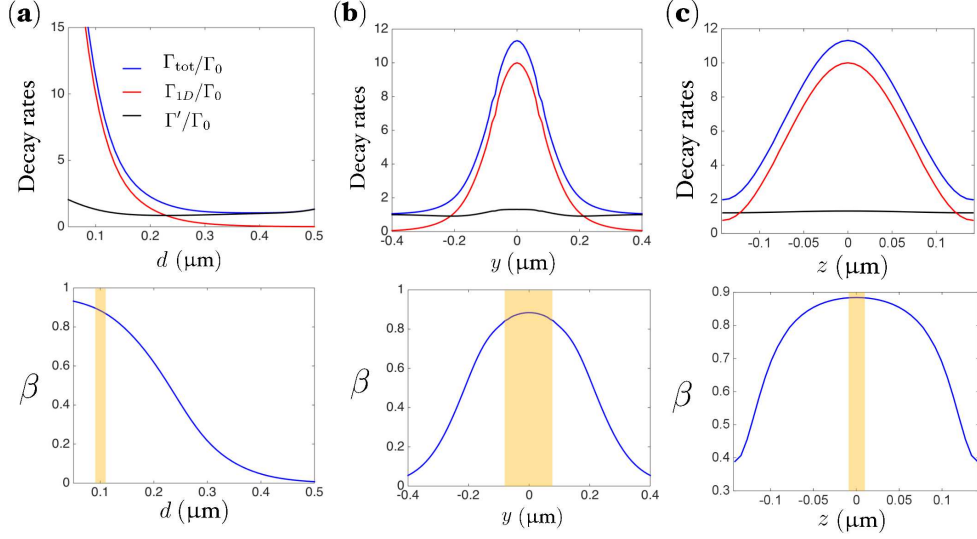


Fig. 6. Decay rates of an atom near the 3D asymmetric comb waveguide. First line: Decay rates Γ_{tot} (blue), Γ_{1D} (red), and Γ' (black) as a function of the distance d from the waveguide for $y = 0$ and $z = 0 \pmod{a}$ (a), as a function of y for $d = 100$ nm and $z = 0 \pmod{a}$ (b), and as a function of z for $d = 100$ nm and $y = 0$ (c). All decay rates are normalized by the decay rate Γ_0 of the atom in vacuum. Second line: Variation of the β factor, $\beta = \Gamma_{1D}/\Gamma_{\text{tot}}$, at the same positions. The yellow areas represent the volume where atoms are likely to be trapped, defined as the positions where the value of the orange potential in Fig. 5 is between $U_{\text{min}} \approx -2.2$ mK and $U_{\text{min}} + 50$ μ K.

434 larger than 30% for $d \leq 100$ nm. This trend of an increase of Γ' close to a dielectric structure is
 435 not general; a previous calculation with the same numerical method has shown a decrease of Γ'
 436 in the near field of a different periodic waveguide [67].

437 6. Conclusion

438 We have proposed a new waveguide geometry, the asymmetric comb, that provides a strong
 439 interaction between trapped atoms and guided photons. An important originality of the structure
 440 is its quartic dispersion relation of the form $\omega - \omega_e \propto -(k - \pi/a)^4$, unique in the context
 441 of waveguide QED. We have demonstrated that this specific form of the dispersion relation
 442 reduces the impact of the inevitable fabrication imperfections. Then, we have shown how cold
 443 Rubidium atoms can be trapped at subwavelength distances ($d = 100$ nm) from the structure by
 444 implementing a two-color optical trap with guided modes that are red and blue detuned with
 445 respect to the atomic transition frequency. Finally, we have completely characterized the decay
 446 of an excited atom in this complex photonic environment by calculating rigorously the decay rate
 447 Γ_{1D} into the guided mode as well as the decay rate Γ' into all other radiative channels. Atoms
 448 inside the trap decay preferentially into the slow mode (group velocity $v_g = c/50$) with a β factor
 449 as high as 0.88.

450 We have considered throughout the article a structure made of GaInP, a semiconductor material
 451 with a high refractive index. This choice is not critical for the design. We have checked that an
 452 asymmetric comb waveguide with a quartic dispersion and a large β factor can also be designed
 453 in a material with a lower refractive index such as silicon nitride or silicon oxide. Regarding
 454 the fabrication, we have conducted preliminary studies (not shown here) that show that the
 455 fabrication of the structure is completely feasible with current fabrication processes. Note that

456 suspended periodic waveguides with symmetrical patterns of similar dimensions have been
457 fabricated in [47]. Two important practical issues are the coupling of light into the slow mode
458 and its propagation loss. Attenuations of 3 dB after propagation over 100 μm have been measured
459 for $n_g = 50$ in a photonic-crystal waveguide [57]. We can reasonably expect lower propagation
460 loss since we have shown that the asymmetric comb waveguide has a larger effective photon
461 mass. Coupling light into the slow mode can be achieved by gradually decreasing the group
462 velocity with an adiabatic tuning of the waveguide geometry. Short (1–2 μm) and efficient (90%
463 efficiency) tapers have been realized with this strategy [68–70].

464 Being able to control the emission with figures of merit as large as $\Gamma_{1D} = 10\Gamma_0$, $\Gamma' = 1.3\Gamma_0$, and
465 $\beta = 0.88$ puts the strong coupling regime of waveguide QED ($\Gamma_{1D}/\Gamma' \gg 1$) within reach. In this
466 important regime, both collective [71–76] and non-linear quantum [2] phenomena are enhanced,
467 meaning that single photon switches [77] or coherent photon storage [17, 18, 29] could be achieved
468 with high efficiencies. Exotic many-body phenomena such as correlated photon transport [21],
469 many-body localization [26], or fermionization of the multiple excited states [78] could also be
470 explored with the asymmetric comb. Finally, let us emphasize that the uncommon dispersion
471 relation of the slow mode significantly alters the range of the photon-mediated interactions
472 for atoms whose transition frequencies lie inside the bandgap, as discussed in Supplement 1.
473 Enlarging the bandwidth of the structure also allows one to probe with reduced distortion the fast
474 dynamics of superradiant emission.

475 **Funding.** This work was supported by the French National Research Agency (ANR) under the NanoStrong
476 Project (ANR-18-CE47-0008) and the Région Ile-de-France (DIM SIRTEQ). This project has also received
477 funding from the European Union Horizon 2020 research and innovation programme under Grant Agreement
478 No. 899275 (DAALI project). A.U. was supported by the European Union (Marie Curie Fellowship
479 SinglePass 101030421).

480 **Disclosures.** The authors declare no conflicts of interest.

481 **Data availability.** Data underlying the results presented in this paper are not publicly available at this
482 time but may be obtained from the authors upon reasonable request.

483 **Supplemental document.** See [Supplement 1](#) for supporting content.

484 References

- 485 1. H. J. Kimble, “The quantum internet,” *Nature* **453**, 1023–1030 (2008).
- 486 2. D. E. Chang, V. Vuletić, and M. D. Lukin, “Quantum nonlinear optics—photon by photon,” *Nat. Photonics* **8**,
487 685–694 (2014).
- 488 3. J.-M. Raimond, M. Brune, and S. Haroche, “Manipulating quantum entanglement with atoms and photons in a cavity,”
489 *Rev. Mod. Phys.* **73**, 565–582 (2001).
- 490 4. A. Reiserer and G. Rempe, “Cavity-based quantum networks with single atoms and optical photons,” *Rev. Mod.*
491 *Phys.* **87**, 1379–1418 (2015).
- 492 5. J.-M. Gérard, B. Sermage, B. Gayral, B. Legrand, E. Costard, and V. Thierry-Mieg, “Enhanced spontaneous emission
493 by quantum boxes in a monolithic optical microcavity,” *Phys. Rev. Lett.* **81**, 1110–1113 (1998).
- 494 6. P. Lodahl, S. Mahmoodian, and S. Stobbe, “Interfacing single photons and single quantum dots with photonic
495 nanostructures,” *Rev. Mod. Phys.* **87**, 347–400 (2015).
- 496 7. T. G. Tiecke, J. D. Thompson, N. P. de Leon, L. R. Liu, V. Vuletić, and M. D. Lukin, “Nanophotonic quantum phase
497 switch with a single atom,” *Nature* **508**, 241–244 (2014).
- 498 8. K. P. Nayak, P. N. Melentiev, M. Morinaga, F. Le Kien, V. I. Balykin, and K. Hakuta, “Optical nanofiber as an
499 efficient tool for manipulating and probing atomic fluorescence,” *Opt. Express* **15**, 5431–5438 (2007).
- 500 9. E. Vetsch, D. Reitz, G. Sagué, R. Schmidt, S. T. Dawkins, and A. Rauschenbeutel, “Optical interface created by
501 laser-cooled atoms trapped in the evanescent field surrounding an optical nanofiber,” *Phys. Rev. Lett.* **104**, 203603
502 (2010).
- 503 10. A. Goban, K. S. Choi, D. J. Alton, D. Ding, C. Lacroûte, M. Pototschnig, T. Thiele, N. P. Stern, and H. J. Kimble,
504 “Demonstration of a state-insensitive, compensated nanofiber trap,” *Phys. Rev. Lett.* **109**, 033603 (2012).
- 505 11. A. Goban, C.-L. Hung, S.-P. Yu, J. D. Hood, J. A. Muniz, J. H. Lee, M. J. Martin, A. C. McClung, K. S. Choi, D. E.
506 Chang, O. Painter, and H. J. Kimble, “Atom–light interactions in photonic crystals,” *Nat. Commun.* **5**, 3808 (2014).
- 507 12. A. Goban, C.-L. Hung, J. D. Hood, S.-P. Yu, J. A. Muniz, O. Painter, and H. J. Kimble, “Superradiance for atoms
508 trapped along a photonic crystal waveguide,” *Phys. Rev. Lett.* **115**, 063601 (2015).

- 509 13. X. Zang, J. Yang, R. Faggiani, C. Gill, P. G. Petrov, J.-P. Hugonin, K. Vynck, S. Bernon, P. Bouyer, V. Boyer, and
510 P. Lalanne, “Interaction between atoms and slow light: a study in waveguide design,” *Phys. Rev. Appl.* **5**, 024003
511 (2016).
- 512 14. D. Roy, C. M. Wilson, and O. Firstenberg, “Colloquium: Strongly interacting photons in one-dimensional continuum,”
513 *Rev. Mod. Phys.* **89**, 021001 (2017).
- 514 15. D. E. Chang, J. S. Douglas, A. González-Tudela, C.-L. Hung, and H. J. Kimble, “Colloquium: Quantum matter built
515 from nanoscopic lattices of atoms and photons,” *Rev. Mod. Phys.* **90**, 031002 (2018).
- 516 16. P. Türschmann, H. Le Jeannic, S. F. Simonsen, H. R. Haakh, S. Götzinger, V. Sandoghdar, P. Lodahl, and N. Rotenberg,
517 “Coherent nonlinear optics of quantum emitters in nanophotonic waveguides,” *Nanophotonics* **8**, 1641–1657 (2019).
- 518 17. B. Gouraud, D. Maxein, A. Nicolas, O. Morin, and J. Laurat, “Demonstration of a memory for tightly guided light in
519 an optical nanofiber,” *Phys. Rev. Lett.* **114**, 180503 (2015).
- 520 18. C. Sayrin, C. Clausen, B. Albrecht, P. Schneeweiss, and A. Rauschenbeutel, “Storage of fiber-guided light in a
521 nanofiber-trapped ensemble of cold atoms,” *Optica* **2**, 353–356 (2015).
- 522 19. N. V. Corzo, J. Raskop, A. Chandra, A. S. Sheremet, B. Gouraud, and J. Laurat, “Waveguide-coupled single collective
523 excitation of atomic arrays,” *Nature* **566**, 359–362 (2019).
- 524 20. S. Mahmoodian, M. Čepulkovskis, S. Das, P. Lodahl, K. Hammerer, and A. S. Sørensen, “Strongly correlated photon
525 transport in waveguide quantum electrodynamics with weakly coupled emitters,” *Phys. Rev. Lett.* **121**, 143601 (2018).
- 526 21. S. Mahmoodian, G. Calajó, D. E. Chang, K. Hammerer, and A. S. Sørensen, “Dynamics of many-body photon bound
527 states in chiral waveguide qed,” *Phys. Rev. X* **10**, 031011 (2020).
- 528 22. A. S. Prasad, J. Hinney, S. Mahmoodian, K. Hammerer, S. Rind, P. Schneeweiss, A. S. Sørensen, J. Volz, and
529 A. Rauschenbeutel, “Correlating photons using the collective nonlinear response of atoms weakly coupled to an
530 optical mode,” *Nat. Photonics* **14**, 719–722 (2020).
- 531 23. J. S. Douglas, H. Habibian, C.-L. Hung, A. V. Gorshkov, H. J. Kimble, and D. E. Chang, “Quantum many-body
532 models with cold atoms coupled to photonic crystals,” *Nat. Photonics* **9**, 326–331 (2015).
- 533 24. G. Calajó and D. E. Chang, “Emergence of solitons from many-body photon bound states in quantum nonlinear
534 media,” *Phys. Rev. Res.* **4**, 023026 (2022).
- 535 25. B. Bakkensen, Y.-X. Zhang, J. Bjerlin, and A. S. Sørensen, “Photonic bound states and scattering resonances in
536 waveguide QED,” arXiv preprint arXiv:2110.06093 (2021).
- 537 26. N. Fayard, L. Henriot, A. Asenjo-Garcia, and D. E. Chang, “Many-body localization in waveguide quantum
538 electrodynamics,” *Phys. Rev. Res.* **3**, 033233 (2021).
- 539 27. H. L. Sørensen, J.-B. Béguin, K. W. Kluge, I. Iakoupov, A. S. Sørensen, J. H. Müller, E. S. Polzik, and J. Appel,
540 “Coherent backscattering of light off one-dimensional atomic strings,” *Phys. Rev. Lett.* **117**, 133604 (2016).
- 541 28. N. V. Corzo, B. Gouraud, A. Chandra, A. Goban, A. S. Sheremet, D. V. Kupriyanov, and J. Laurat, “Large bragg
542 reflection from one-dimensional chains of trapped atoms near a nanoscale waveguide,” *Phys. Rev. Lett.* **117**, 133603
543 (2016).
- 544 29. P. Solano, P. Barberis-Blostein, F. K. Fatemi, L. A. Orozco, and S. L. Rolston, “Super-radiance reveals infinite-range
545 dipole interactions through a nanofiber,” *Nat. Commun.* **8**, 1857 (2017).
- 546 30. V. I. Balykin, K. Hakuta, F. Le Kien, J. Q. Liang, and M. Morinaga, “Atom trapping and guiding with a subwavelength-
547 diameter optical fiber,” *Phys. Rev. A* **70**, 011401 (2004).
- 548 31. F. Le Kien, V. I. Balykin, and K. Hakuta, “Atom trap and waveguide using a two-color evanescent light field around a
549 subwavelength-diameter optical fiber,” *Phys. Rev. A* **70**, 063403 (2004).
- 550 32. T. Nieddu, V. Gokhroo, and S. N. Chormaic, “Optical nanofibres and neutral atoms,” *J. Opt.* **18**, 053001 (2016).
- 551 33. P. Forn-Díaz, L. Lamata, E. Rico, J. Kono, and E. Solano, “Ultrastrong coupling regimes of light-matter interaction,”
552 *Rev. Mod. Phys.* **91**, 025005 (2019).
- 553 34. A. S. Sheremet, M. I. Petrov, I. V. Iorsh, A. V. Poshakinskiy, and A. N. Poddubny, “Waveguide quantum electrodynamics:
554 collective radiance and photon-photon correlations,” arXiv preprint arXiv:2103.06824 (2021).
- 555 35. G. Lecamp, P. Lalanne, and J.-P. Hugonin, “Very large spontaneous-emission β factors in photonic-crystal waveguides,”
556 *Phys. Rev. Lett.* **99**, 023902 (2007).
- 557 36. J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, *Molding the flow of light* (Princeton University,
558 2008), chap. 7.
- 559 37. C.-L. Hung, S. M. Meenehan, D. E. Chang, O. Painter, and H. J. Kimble, “Trapped atoms in one-dimensional
560 photonic crystals,” *New J. Phys.* **15**, 083026 (2013).
- 561 38. S.-P. Yu, J. D. Hood, J. A. Muniz, M. J. Martin, R. Norte, C.-L. Hung, S. M. Meenehan, J. D. Cohen, O. Painter, and
562 H. J. Kimble, “Nanowire photonic crystal waveguides for single-atom trapping and strong light-matter interactions,”
563 *Appl. Phys. Lett.* **104**, 111103 (2014).
- 564 39. J.-B. Béguin, A. P. Burgers, X. Luan, Z. Qin, S. P. Yu, and H. J. Kimble, “Advanced apparatus for the integration of
565 nanophotonics and cold atoms,” *Optica* **7**, 1–2 (2020).
- 566 40. R. Faggiani, A. Baron, X. Zang, L. Lalouat, S. A. Schulz, B. O’Regan, K. Vynck, B. Cluzel, F. de Fornel, T. F.
567 Krauss, and P. Lalanne, “Lower bound for the spatial extent of localized modes in photonic crystal waveguides with
568 small random imperfections,” *Sci. Reports* **6**, 27037 (2016).
- 569 41. M. Loncar, D. Nedeljkovic, T. Doll, J. Vuckovic, A. Scherer, and T. Pearsall, “Waveguiding in planar photonic
570 crystals,” *Appl. Phys. Lett.* **77**, 1937–1939 (2000).
- 571 42. M. Notomi, K. Yamada, A. Shinya, J. Takahashi, C. Takahashi, and I. Yokohama, “Extremely large group-velocity

- 572 dispersion of line-defect waveguides in photonic crystal slabs,” *Phys. Rev. Lett.* **87**, 253902 (2001).
- 573 43. M. Gnan, W. C. L. Hopman, G. Bellanca, R. M. de Ridder, R. M. De La Rue, and P. Bassi, “Closure of the stop-band
- 574 in photonic wire bragg gratings,” *Opt. Express* **17**, 8830–8842 (2009).
- 575 44. P. J. Bock, P. Cheben, J. H. Schmid, J. Lapointe, A. Delage, S. Janz, G. C. Aers, D.-X. Xu, A. Densmore, and T. J.
- 576 Hall, “Subwavelength grating periodic structures in silicon-on-insulator: a new type of microphotonic waveguide,”
- 577 *Opt. Express* **18**, 20251–20262 (2010).
- 578 45. M. Ibrahim, J. H. Schmid, A. Aleali, P. Cheben, J. Lapointe, S. Janz, P. J. Bock, A. Densmore, A. Lamontagne,
- 579 R. Ma, D.-X. Xu, and W. N. Ye, “Athermal silicon waveguides with bridged subwavelength gratings for TE and TM
- 580 polarizations,” *Opt. Express* **20**, 18356–18361 (2012).
- 581 46. J. S. Penadés, C. Alonso-Ramos, A. Z. Khokhar, M. Nedeljkovic, L. A. Boodhoo, A. Ortega-Moñux, I. Molina-
- 582 Fernández, P. Cheben, and G. Z. Mashanovich, “Suspended soi waveguide with sub-wavelength grating cladding for
- 583 mid-infrared,” *Opt. Lett.* **39**, 5661–5664 (2014).
- 584 47. C. Alonso-Ramos, X. Le Roux, J. Zhang, D. Benedikovic, V. Vakarin, E. Durán-Valdeiglesias, D. Oser, D. Pérez-
- 585 Galacho, F. Mazeas, L. Labonté, S. Tanzilli, E. Cassan, D. Marris-Morini, P. Cheben, and L. Vivien, “Diffraction-less
- 586 propagation beyond the sub-wavelength regime: a new type of nanophotonic waveguide,” *Sci. Reports* **9**, 5347
- 587 (2019).
- 588 48. H. S. Nguyen, F. Dubois, T. Deschamps, S. Cueff, A. Pardon, J.-L. Leclercq, C. Seassal, X. Letartre, and P. Viktorovitch,
- 589 “Symmetry breaking in photonic crystals: On-demand dispersion from flatband to dirac cones,” *Phys. Rev. Lett.* **120**,
- 590 066102 (2018).
- 591 49. G. Lecamp, J.-P. Hugonin, and P. Lalanne, “Theoretical and computational concepts for periodic optical waveguides,”
- 592 *Opt. Express* **15**, 11042–11060 (2007).
- 593 50. E. Silberstein, P. Lalanne, J.-P. Hugonin, and Q. Cao, “Use of grating theories in integrated optics,” *J. Opt. Soc. Am.*
- 594 **A 18**, 2865–2875 (2001).
- 595 51. J.-P. Hugonin and P. Lalanne, “Perfectly matched layers as nonlinear coordinate transforms: a generalized
- 596 formalization,” *J. Opt. Soc. Am. A* **22**, 1844–1849 (2005).
- 597 52. S. Hughes, L. Ramunno, J. F. Young, and J. E. Sipe, “Extrinsic optical scattering loss in photonic crystal waveguides:
- 598 Role of fabrication disorder and photon group velocity,” *Phys. Rev. Lett.* **94**, 033903 (2005).
- 599 53. J. Topolancik, B. Ilic, and F. Vollmer, “Experimental observation of strong photon localization in disordered photonic
- 600 crystal waveguides,” *Phys. Rev. Lett.* **99**, 253901 (2007).
- 601 54. S. Mazoyer, J.-P. Hugonin, and P. Lalanne, “Disorder-induced multiple scattering in photonic-crystal waveguides,”
- 602 *Phys. Rev. Lett.* **103**, 063903 (2009).
- 603 55. M. Patterson, S. Hughes, S. Combrié, N.-V.-Q. Tran, A. De Rossi, R. Gabet, and Y. Jaouën, “Disorder-induced
- 604 coherent scattering in slow-light photonic crystal waveguides,” *Phys. Rev. Lett.* **102**, 253903 (2009).
- 605 56. P. D. García, S. Smolka, S. Stobbe, and P. Lodahl, “Density of states controls Anderson localization in disordered
- 606 photonic crystal waveguides,” *Phys. Rev. B* **82**, 165103 (2010).
- 607 57. S. Mazoyer, P. Lalanne, J.-C. Rodier, J.-P. Hugonin, M. Spasenović, L. Kuipers, D. M. Beggs, and T. F. Krauss,
- 608 “Statistical fluctuations of transmission in slow light photonic-crystal waveguides,” *Opt. Express* **18**, 14654–14663
- 609 (2010).
- 610 58. A. P. Burgers, L. S. Peng, J. A. Muniz, A. C. McClung, M. J. Martin, and H. J. Kimble, “Clocked atom delivery to a
- 611 photonic crystal waveguide,” *Proc. National Acad. Sci.* **116**, 456–465 (2019).
- 612 59. J.-B. Beguin, J. Laurat, X. Luan, A. P. Burgers, Z. Qin, and H. J. Kimble, “Reduced volume and reflection for bright
- 613 optical tweezers with radial Laguerre–Gauss beams,” *Proc. National Acad. Sci.* **117**, 26109–26117 (2020).
- 614 60. J. D. Hood, A. Goban, A. Asenjo-Garcia, M. Lu, S.-P. Yu, D. E. Chang, and H. J. Kimble, “Atom–atom interactions
- 615 around the band edge of a photonic crystal waveguide,” *Proc. National Acad. Sci.* **113**, 10507–10512 (2016).
- 616 61. H. B. G. Casimir and D. Polder, “The influence of retardation on the London-van der Waals forces,” *Phys. Rev.* **73**,
- 617 360–372 (1948).
- 618 62. S. Y. Buhmann and D.-G. Welsch, “Dispersion forces in macroscopic quantum electrodynamics,” *Prog. Quantum*
- 619 *Electron.* **31**, 51–130 (2007).
- 620 63. A. O. Caride, G. L. Klimchitskaya, V. M. Mostepanenko, and S. I. Zhanette, “Dependences of the van der Waals
- 621 atom-wall interaction on atomic and material properties,” *Phys. Rev. A* **71**, 042901 (2005).
- 622 64. M. Schubert, V. Gottschalch, C. M. Herzinger, H. Yao, P. G. Snyder, and J. A. Woollam, “Optical constants of
- 623 $\text{Ga}_x\text{In}_{1-x}\text{P}$ lattice matched to GaAs,” *J. Appl. Phys.* **77**, 3416–3419 (1995).
- 624 65. J. Berroir, A. Bouscal, A. Urvoy, T. Ray, and J. Laurat, “Nanotrappy: An open-source versatile package for cold-atom
- 625 trapping close to nanostructures,” *Phys. Rev. Res.* **4**, 013079 (2022).
- 626 66. L. Novotny and B. Hecht, *Principles of Nano-Optics* (Cambridge University, 2006), 2nd ed.
- 627 67. X. Zang and P. Lalanne, “Theoretical treatment of the interaction between two-level atoms and periodic waveguides,”
- 628 *Opt. Lett.* **40**, 3869–3872 (2015).
- 629 68. J.-P. Hugonin, P. Lalanne, T. P. White, and T. F. Krauss, “Coupling into slow-mode photonic crystal waveguides,”
- 630 *Opt. Lett.* **32**, 2638–2640 (2007).
- 631 69. P. Pottier, M. Gnan, and R. M. De La Rue, “Efficient coupling into slow-light photonic crystal channel guides using
- 632 photonic crystal tapers,” *Opt. Express* **15**, 6569–6575 (2007).
- 633 70. C. M. de Sterke, J. Walker, K. B. Dossou, and L. C. Botten, “Efficient slow light coupling into photonic crystals,”
- 634 *Opt. Express* **15**, 10984–10990 (2007).

- 635 71. D. Plankensteiner, L. Ostermann, H. Ritsch, and C. Genes, “Selective protected state preparation of coupled dissipative
636 quantum emitters,” *Sci. Reports* **5**, 16231 (2015).
- 637 72. A. Asenjo-Garcia, M. Moreno-Cardoner, A. Albrecht, H. J. Kimble, and D. E. Chang, “Exponential improvement in
638 photon storage fidelities using subradiance and “selective radiance” in atomic arrays,” *Phys. Rev. X* **7**, 031024 (2017).
- 639 73. D. F. Kornovan, N. V. Corzo, J. Laurat, and A. S. Sheremet, “Extremely subradiant states in a periodic one-dimensional
640 atomic array,” *Phys. Rev. A* **100**, 063832 (2019).
- 641 74. I. Shlesinger, P. Senellart, L. Lanco, and J.-J. Greffet, “Time-frequency encoded single-photon generation and
642 broadband single-photon storage with a tunable subradiant state,” *Optica* **8**, 95–105 (2021).
- 643 75. G. Ferioli, A. Glicenstein, L. Henriot, I. Ferrier-Barbut, and A. Browaeys, “Storage and release of subradiant
644 excitations in a dense atomic cloud,” *Phys. Rev. X* **11**, 021031 (2021).
- 645 76. M. Reitz, C. Sommer, and C. Genes, “Cooperative quantum phenomena in light-matter platforms,” *PRX Quantum* **3**,
646 010201 (2022).
- 647 77. D. E. Chang, A. S. Sørensen, E. A. Demler, and M. D. Lukin, “A single-photon transistor using nanoscale surface
648 plasmons,” *Nat. Phys.* **3**, 807–812 (2007).
- 649 78. A. Albrecht, L. Henriot, A. Asenjo-Garcia, P. B. Dieterle, O. Painter, and D. E. Chang, “Subradiant states of quantum
650 bits coupled to a one-dimensional waveguide,” *New J. Phys.* **21**, 025003 (2019).